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Appendix

Elastic region

The Lamé analysis for a thick-walled closed-ended cylinder gives radial, tangential and axial stresses which can be interpreted as being equivalent to a shear stress of

$$\tau = \frac{P}{K^2-1}\frac{R_o^2}{r^2}$$

together with a hydrostatic or volumetric tensile stress of

$$\sigma_m = \frac{P}{K^2-1} \; .$$

Here P is the internal pressure, $K = R_o/R_i$ is the ratio of the outer to the inner radius, and r is any intermediate radius.

The maximum shear stress occurs at the bore where $r = R_i$ and yield will occur when

$$\tau_{yield} = \frac{K^2}{K^2-1}P_{yield}$$

or

$$P_{yield} = \frac{(K^2-1)}{K^2}\tau_{yield} \; .$$

P_{yield} is the pressure at initial yield. The diametral (hoop) strain in the elastic region will be

$$(\epsilon_\theta)_r = R_o = \frac{P}{E(K^2-1)}(2-\nu)$$

where E is Young's modulus and ν is Poisson's ratio.

Collapse pressure

If the material of which the cylinder is made yields at a constant shear stress then, when yield has spread to the outer diameter, straining will continue at constant pressure until the material begins to strain harden. The pressure required to just cause complete yielding is known as the collapse pressure, though it is sometimes wrongly termed ballooning pressure, which might give the impression that local instability or local bulging was occurring, which in fact does not occur until the ultimate pressure is exceeded. The collapse pressure can be shown to be

$$P_c = \tau_{PY}\ln K^2$$

where τ_{PY} is the shear stress at plastic yield.

Ultimate pressure

When the pressure is raised above the collapse pressure the material is becoming strain hardened while the diameter ratio K is decreasing due to wall thinning. Finally, the weakening effect due to reduction of K exceeds the strengthening effect due to work hardening, and the equilibrium pressure passes through a maximum which is termed the ultimate pressure. The analysis of this regime is dealt with fully by Crossland, Jorgensen and Bones (1958), Crossland (1964) and Manning (1945) and space does not permit its consideration here.

Several empirical equations have been suggested including that due to Faupel (1956) which is expressed in the form

$$P_{\mathrm{ult}} = \frac{2\sigma_{\mathrm{PY}}}{\sqrt{3}}\left(2-\frac{\sigma_{\mathrm{PY}}}{\sigma_{\mathrm{ult}}}\right)\ln K$$

and the mean diameter formula

$$P_{\mathrm{ult}} = 2\sigma_{\mathrm{ult}}\frac{K-1}{K+1} .$$

Here P_{ult} is the ultimate pressure, and σ_{ult} the ultimate tensile stress.

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